PROPOSITIONAL LOGIC – ARGUMENTS

1. Write down the argument symbolically i.e. find its structure (translate it to the language of propositional logic).
2. Prepare the truth table so that each premise and the conclusion has a separate column.
3. Fill in the table.
4. Look for a row in which all the premises are true and the conclusion is false (the counterexample row).
5. If there is no such a row, the argument structure, and hence the argument itself, is formally correct (valid).
6. If such a row exists, the structure is formally incorrect (invalid). WARNING: There is no guarantee that the original argument is invalid (it may be a substitution of some other, this time valid, structure).

<u>Example 1</u>

If you pass the exam, you will get a car.
<u>You will not pass the exam.</u>
So you will not get a car.

$$\begin{array}{c} p \to q \\ \underline{\neg p} \\ \neg q \end{array}$$

p	q	$p \to q$	$\neg p$	$\neg q$
1	1	1	0	0
1	0	0	0	1
<u>0</u>	**<u>1</u>**	**<u>1</u>**	**<u>1</u>**	**<u>0</u>**
0	0	1	1	1

Argument structure is invalid (formally incorrect). In this case original argument (with exam and car) is also invalid.

Example 2

$$\begin{array}{c} p \lor \neg q \\ \underline{\quad q \quad} \\ \mathrm{p} \end{array}$$

p	q	p	$\lor$	$\neg$	q	q	p
1	1		1	0		1	1
1	0		1	1		0	1
0	1		0	0		1	0
0	0		1	1		0	0

Argument structure is valid (formally correct). Every argument that is a substitution of this structure is also valid (formally correct).

Example 3 (modus tollendo tollens)

$$\begin{array}{c} p \rightarrow q \\ \underline{\neg q} \\ \neg p \end{array}$$

p	q	p	$\rightarrow$	q	$\neg$	q	$\neg$	p
1	1		1		0		0	
1	0		0		1		0	
0	1		1		0		1	
0	0		1		1		1	

Argument structure is valid (formally correct). Every argument that is a substitution of this structure is also valid (formally correct).

Example 4 (modus ponendo ponens)

$$\begin{array}{c} p \to q \\ \underline{\;p\;} \\ q \end{array}$$

p	q	$p \to q$	p	q
1	1	1	1	1
1	0	0	1	0
0	1	1	0	1
0	0	1	0	0

Argument structure is valid (formally correct). Every argument that is a substitution of this structure is also valid (formally correct).

Example 5

$$\begin{array}{c} p \to q \\ q \to r \\ \underline{\;\neg r\;} \\ \neg p \end{array}$$

p	q	r	$p \to q$	$q \to r$	$\neg r$	$\neg p$
1	1	1	1	1	0	0
1	1	0	1	0	1	0
1	0	1	0	1	0	0
1	0	0	0	1	1	0
0	1	1	1	1	0	1
0	1	0	1	0	1	1
0	0	1	1	1	0	1
0	0	0	1	1	1	1

Argument structure is valid (formally correct). Every argument that is a substitution of this structure is also valid (formally correct).

Example 6 (Destruction of the Library of Alexandria)

In 642 AD Caliph Omar I burned the Library arguing: 'If those books are in agreement with the Quran, we have no need of them; and if these are opposed to the Quran, they are harmful.'.

1. Books are in agreement with the Quran, or are not in agreement (are opposed) with the Quran.
2. If books are in agreement with the Quran they are useless.
3. If books are opposed to the Quran they are harmful.
4. If books are harmful or useless they should be destroyed.
5. The books (Library) should be destroyed.

$$p \vee \neg p$$
$$p \rightarrow \neg q$$
$$\neg p \rightarrow r$$
$$\underline{(r \vee \neg q) \rightarrow s}$$
$$s$$

p - Books are in agreement with the Quran

q – Books are useful

r – Books are harmful

s – Books should be destroyed

p	q	r	s	p	$\vee$	$\neg$	p	p	$\rightarrow$	$\neg$	q	$\neg$	p	$\rightarrow$	r	$(r$	$\vee$	$\neg$	$q)$	$\rightarrow$	s	s
1	1	1	1		1	0			0	0		0		1			1	0		1		1
1	1	1	0		1	0			0	0		0		1			1	0		0		0
1	1	0	1		1	0			0	0		0		1			0	0		1		1
1	1	0	0		1	0			0	0		0		1			0	0		1		0
1	0	1	1		1	0			1	1		0		1			1	1		1		1
1	0	1	0		1	0			1	1		0		1			1	1		0		0
1	0	0	1		1	0			1	1		0		1			1	1		1		1
1	0	0	0		1	0			1	1		0		1			1	1		0		0
0	1	1	1		1	1			1	0		1		1			1	0		1		1
0	1	1	0		1	1			1	0		1		1			1	0		0		0
0	1	0	1		1	1			1	0		1		0			0	0		1		1
0	1	0	0		1	1			1	0		1		0			0	0		1		0
0	0	1	1		1	1			1	1		1		1			1	1		1		1
0	0	1	0		1	1			1	1		1		1			1	1		0		0
0	0	0	1		1	1			1	1		1		0			1	1		1		1
0	0	0	0		1	1			1	1		1		0			1	1		0		0

Argument structure is valid (formally correct). Every argument that is a substitution of this structure is also valid (formally correct). Therefore Caliph Omar’s argument is valid (formally correct). Hovewer it is not materially correct (some premises are false) and hence not sound.

TAUTOLOGIES

Every formula of propositional logic, which is always true (i.e. in the case of every possible combination of true values of variables) is called tautology (universal truth).

REMARK

Tautologies are universal truths, i.e. they are always true. Therefore, they say nothing interesting about the world.

Example

$$p \vee \neg p$$

It is going to rain tomorrow or it is not going to rain tomorrow.

Example

„$[p \to (q \to r)] \leftrightarrow [(p \wedge q) \to r]$”

	q	r	$[p$	$\to$	$(q$	$\to$	$r)]$	$\leftrightarrow$	$[(p$	$\wedge$	$q)$	$\to$	$r]$
1	1	1		1		1		**1**		1		1	
1	1	0		0		0		**1**		1		0	
1	0	1		1		1		**1**		0		1	
1	0	0		1		1		**1**		0		1	
0	1	1		1		1		**1**		0		1	
0	1	0		1		0		**1**		0		1	
0	0	1		1		1		**1**		0		1	
0	0	0		1		1		**1**		0		1	