LOGICAL ANALYSIS (PROPOSITIONAL LOGIC)

The purpose of logical analysis of a given expression is to present the logical structure of this expression, by translating (paraphrasing) this expression into the artificial language of a certain logical calculus.

REMARK !!!
In the remainder, we will deal with the logical analysis of sentences and we will conduct it through the decomposition of sentences into the simplest sentences possible and connecting them by sentence-forming functors (i.e. to the language of propositional logic).

ALPHABET OF PROPOSITIONAL LOGIC:

1. Sentence variables (sentence letters): $p, q, r, s\ldots$
2. Constants (truth functors, sentence-forming functors):
   $\neg\ (\sim), \quad \wedge, \quad \vee, \quad \rightarrow, \quad \leftrightarrow$
3. Auxiliary marks: *(,),{,}*

EXPRESSIONS OF PROPOSITIONAL LOGIC:

Every finite string of symbols from the alphabet of propositional logic is an expression of propositional logic.

$$p \vee q, \qquad pq \rightarrow r, \qquad (p \vee q) \leftrightarrow (r \wedge s)$$
$$(()\{(), \qquad \vee\leftrightarrow\wedge$$

up, put

WELL FORMED FORMULAS (WFF) OF PROPOSITIONAL LOGIC

1. Every single sentence letter is a WFF of propositional logic
2. If α and β are WFF of propositional logic, so are also:
   $(\neg\alpha), \quad (\alpha \wedge \beta), \quad (\alpha \vee \beta), \quad (\alpha \rightarrow \beta), \quad (\alpha \leftrightarrow \beta)$

REMARK 1
1 – truth (sometimes denoted by „T")
0 – falsehood (sometimes denoted by „F")

REMARK 2
Each assignment of logical values to all sentence variables occurring in the expression (and thus every row in the table) is a model of this expression (a set of possible worlds, a possible scenario, an interpretation).

REMARK 3
The number of models (sets of possible worlds, possible scenarios, interpretations, table rows) for a given expression amounts to 2^n, where n is the number of sentence variables in the expression.

expression with three variables: p_1, p_2, p_3 – $2^3 = 8$ models
expression with four variables: p_1, p_2, p_3, p_4 – $2^4 = 16$ models
expression with five variables: p_1, p_2, p_3, p_4, p_5 – $2^5 = 32$ models
…

TRUTH FUNCTORS

The classic approach distinguishes five basic truth functors:

1. negtion; $\neg p$ ($\sim p$)
2. conjunction; $p \wedge q$
3. disjunction; $p \vee q$
4. implication; $p \rightarrow q$
5. equivalence; $p \leftrightarrow q$

NEGATION

$\neg p$ ($\sim p$)

p	$\neg p$
1	0
0	1

It is not true, that *p*.
It is false, that *p*.

Examples of other ways to express negation in natural language:

1. I'm not a circus performer.
   ¬ I'm a circus performer. (It is not true, that I'm a circus performer.)
2. Paul did nothing.
   ¬ Paul did something.
3. Denis Villeneuve has never been to Athens.
   ¬ Denis Villeneuve has been to Athens.

ATTENTION !!!

I have not stopped smoking cigarettes.
CAN NOT be translated as:
¬ I have stopped smoking cigarettes.

CONJUNCTION

$p \wedge q$

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

p and q.

REMARK Two interpretations of conjunction are available.
Example: Alfred Hitchcock went outside and noticed that 10 cm of snow had fallen.

According to STRONG interpretation of conjunction (i, $\wedge$) there is a time order (first he left, later saw).
WEAK interpretation doesn't assume so.
We employ WEAK interpretation.

Examples of other ways to express conjunction in natural language:

1. We went to cinema, although it was late.
   We went to cinema $\wedge$ It was late.
2. Carrots are a rich source of vitamin A and K.
   Carrots are a rich source of vitamin A $\wedge$ Carrots are a rich source of vitamin K.
3. This comedy was primitive but funny
   This comedy was primitive $\wedge$ This comedy was funny.
4. Neither me, nor my wife enjoyed the movie.
   I didn't enjoy the movie. $\wedge$ My wife didn't enjoy the movie.
5. Witness' testimony was both unbelievable and disgusting.
   Witness' testimony was unbelievable. $\wedge$ Witness' testimony was disgusting.

ATTENTION !!!

1. Cracow and Lublin are about 300 km apart.
   NOT
   Cracow is about 300 km apart $\wedge$ Lublin is about 300 km apart.
2. The police caught and punished 30 drivers with fines.
   NOT
   The police caught 30 drivers. $\wedge$ The police punished 30 drivers with fines.

DISJUNCTION

$p \vee q$

p	q	$p \vee q$
1	1	1
1	0	1
0	1	1
0	0	0

p or q.

REMARK Two interpretations of disjunction are available.
Example: In the evening we will go to the theatre or we will be swimming in the pool.
According to STRONG interpretation of disjunction (or, $\vee$) only one of two possibilities will happen (either…, or…).
WEAK interpretation doesn't assume so.
We employ WEAK interpretation.

Examples of other ways to express disjunction in natural language:

1. He is stupid or bribed.
   He is stupid $\vee$ He is bribed.

ATTENTION !!!

Mrs. Kowalska can teach you mathematics or physics.

NOT

Mrs. Kowalska can teach you mathematics $\vee$ Mrs. Kowalska can teach you physics.

RATHER

Mrs. Kowalska can teach you mathematics $\wedge$ Mrs. Kowalska can teach you physics.

IMPLICATION

$p \rightarrow q$

p	q	$p \rightarrow q$
1	1	1
1	0	0
0	1	1
0	0	1

If p, then q .
p implies q.

Example: If the paper turns red, then examined solution is acid.
The above statement suggests some relationship between the color of the paper and the nature of the solution and the inverse relationship (if it doesn't turn red, it is not an acid).
NONE of this information is implied by implication.

Examples of other ways to express implication in natural language:

1. Unless he lose his job, he will pay off his loan in half a year.
   He will not lose his job. $\rightarrow$ He will pay off his loan in half a year.
   He will lose his job. $\vee$ He will pay off his loan in half a year.
2. We'll get to the cinema on time, provided we leave in 5 minutes.
   We leave in 5 minutes. $\rightarrow$ We'll get to the cinema on time.
3. Duda will win the next presidential election only if he will get the votes of farmers.
   Duda will win the next presidential election $\rightarrow$ Duda will get the votes of farmers.

ATTENTION !!!

If you like to wash your hands, the bathroom is on the right.

NOT

You like to wash your hands. $\rightarrow$ The bathroom is on the right.

EQUIVALENCE

$p \leftrightarrow q$

p	q	$p \leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1

p if and only if q.
p precisely if q.

Examples of other ways to express equivalence in natural language:

1. The parallelogram is a diamond just if its diagonals intersect at a right angle.
   The parallelogram is a diamond. ↔ Parallelogram's diagonals intersect at a right angle.

EXAMPLE

If you saw all Star Wars' episodes, then you are a fan of the franchise and you are going to watch a new episode, given that it will be released.

You saw all Star Wars' episodes. → You are a fan of the franchise and you are going to watch a new episode, given that it will be released.

You saw all Star Wars' episodes. → (You are a fan of the Star Wars franchise ∧ You are going to watch a new Star Wars episode, given that it will be released)

You saw all Star Wars' episodes. → (You are a fan of the Star Wars franchise ∧ (New Star Wars episode will be released → You are going to watch a new Star Wars episode)).

$(p_1) \rightarrow (p_2 \land (p_3 \rightarrow p_4))$.

p_1 – You saw all Star Wars' episodes.
p_2 – You are a fan of the Star Wars franchise
p_3 – New Star Wars episode will be released
p_4 – You are going to watch a new Star Wars episode