LANGUAGE

Why language?
- to communicate thoughts and knowledge (especially indirect)
- to conduct reasonings

What is a language?
- dictionary (collection of expressions, simple symbols)
- rules (especially of creating complex expressions out of simple ones).

Types of languages (by origin):

1. Natural, ordinary (English, Polish, French, Spanish etc.)
2. Artificial, constructed, planned (esperanto, musical notation, mathematical notation)

MEANING

Definition: The meaning of some expression is the way of understanding that expression determined by the rules of a given language.

Types of expressions (by kind of meaning)

1. Logical constants (quantifiers - $\forall$, $\exists$, statement-forming functors - $\land$, $\lor$, $\rightarrow$, $\leftrightarrow$)
2. Variables – have defined syntactical category, but no definite reference to reality ($3 + x = 5$)
3. Auxiliary marks (brackets, full stops)

SYNTACTICAL CATEGORIES
(basic types of expressions)

Definition: Expressions E_1 and E_2 belong to the same **syntactical category** of the language L iff (if and only if) after replacing expression E_1 by expression E_2 in a well formed expression of language L we again obtain a well formed expression of language L.

During this course we will discuss three syntactical categories. These are:
1. SENTENCES (STATEMENTS)
2. NAMES (TERMS)
3. FUNCTORS (OPERATORS)

SENTENCES (STATEMENTS)

Definition: **Sentence** (in the sense of logic) is an expression, which is true or false.

- Truth and falsity are called <u>logical values.</u>

- Sentence is an expression that states something.

- Sentence in the sense of logic need to be a declarative sentence (questions and orders/commands have no logical values, hence they are not sentences in the sense of logic).

ATTENTION
Not every declarative sentence is a sentence in the sense of logic (e.g. He is a teacher; ambiguous sentences).
Declarative sentences are also used in functions other than the function of stating certain facts:
- persuasive functions (e.g. "No reasonable person buys this newspaper")
- performative functions (e.g. "I baptize you", "I give you the name *Titanic*")

The meaning of a sentence is called a <u>proposition</u>.

Different sentences may have the same proposition (e.g. „dwa plus dwa równa się cztery"; „two plus two equals four"; „2+2=4")

BUT

one sentence should not have more than one proposition (if one sentence has more than one proposition, then it is an ambiguous sentence and as such it is NOT a sentence in the sense of logic).

<u>Types of sentences</u>

By presence of negation:
    1.   Affirmative (e.g. "Earth is a planet")
    2.   Negative (e.g. "Sun is not a planet")

By its range (extension):
    1.   Singular – of one object (e.g. "Earth is a planet")
    2.   Particular – of a subset of some set (e.g. "Some people are Chinese")
    3.   Universal (general) – of a whole set (e.g. "Every human is a mammal")

<u>SENTENTIAL FUNCTION (OPEN FORMULA)</u>

Definition: <u>Sentential function</u> is an expression which contains at least one free variable (not bound by quantifier; such that can be substituted by some other expression) and which is transformed into a sentence by substituting free variables with constants or by binding free variables with quantifiers.

Examples:
$$x + 14 = 20$$
He was the Emperor of the French (in the sense "x was the Emperor of the French")

Sentential function can be transformed into a sentence in two ways:
    1.   by substituting every free variable with some constant,
    2.   by binding every free variable with some quantifier.

Quantifiers
- $\forall_x$ - for all x (for every x)
- $\exists_x$ - there exists x (for some x)

Examples:

$$x + 14 = 20 \text{ (sentential function, has no logical value)}$$
$$\forall_x \; x + 14 = 20 \text{ (false sentence)}$$
$$\exists_x \; x + 14 = 20 \text{ (true sentence)}$$

He was the Emperor of the French. (sentential function, has no logical value)
Everyone was the Emperor of the French. (false sentence)
Someone was the Emperor of the French. (true sentence)

Definition: Object O <u>satisfies</u> a sentential function F iff by substituting the free variable of the propositional function F with the name of the object O function F is transformed into a true sentence.

What (who) satisfies sentential function: "He was the Emperor of the French" ("x was the Emperor of the French")? Napoleon Bonaparte does.

What satisfies sentential function: "x + 14 = 20"? Number „6" does.